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# GAUSS AND THE EIGHT QUEENS PROBLEM: A STUDY IN MINIATURE OF THE PROPAGATION OF HISTORICAL ERROR

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*Summaries*

*An 1874 article by J. W. L. Glaisher asserted that the eight queens problem of recreational mathematics originated in 1850 with Franz Nauck proposing it to Gauss, who then gave the complete solution. In fact the problem was first proposed two years earlier by Max Bezzel, proposed again by Nauck in a newspaper Gauss happened to read, and only partially solved by Gauss in a casual attempt. Glaisher had access to an accurate account of the history in German but perhaps could not read the language well; the error subsequently spread through the recreational mathematics literature.*

*En 1874, J. W. L. Glaisher affirmait dans un article que le problème des huit reines des mathématiques récréatives avait été énoncé pour la première fois par Franz Nauck en 1850 lorsque ce dernier le proposa à Gauss qui en aurait donné alors une solution complète. En fait, Max Bezzel avait déjà proposé ce problème deux années auparavant. Il fut à nouveau énoncé par Nauck dans un journal que Gauss vint à lire. Celui-ci le solutionna partiellement dans une tentative informelle. Glaisher avait accès à une description juste de ces faits en allemand, mais peut-être ne pouvait-il pas lire adéquatement cette langue. Il s'ensuivit que l'erreur se répendit dans toute la littérature sur les mathématiques récréatives.*

## INTRODUCTION

The bicentenary of the birth of Gauss furnishes a suitable occasion for the review and appreciation of his work, and many papers and addresses are being given celebrating various dimensions of his achievements. This paper will focus on another realm altogether, in investigating how a certain famous result in recreational mathematics came to be attributed to Gauss despite his own disclaimer. The explanation draws a moral or two for the mathematician who would spice up the exposition of new results with the history of previous efforts.

## THE PROBLEM AND ITS EARLY HISTORY

The eight queens problem can be formulated in a succinct fashion. The task is to place eight chess queens on an 8x8 chessboard so that none of them lies in the field of attack of any of the others. In other words, no two may lie in the same row, column, or diagonal. There are a total of 92 solutions, which fall into 12 classes (11 of 8 solutions and 1 of 4) when symmetric solutions are identified.

An accurate account of the early history of attempts at solutions, including Gauss' involvement, is as follows:

The first known proposal of the problem was by "Schachfreund" in the September 1848 issue of *Schachzeitung*, a famous chess newspaper [1848]. We know from Lange [1862, 30] that "Schachfreund" in fact was Max Bezzel, a German chessplayer. Two solutions were published in the January 1849 issue, and a total of 40 solutions appeared in *Schachzeitung* over the years 1849 to 1854.

The problem was proposed again, to a different audience--possibly independently--by Franz Nauck, in the more widely read *Illustrirte Zeitung* of Leipzig, in its issue of June 1, 1850 [1850a]. Nauck also included a subproblem, to find all solutions with queens at positions b4 and d5. Nauck asserted in the June 29 [1850b] issue that there are 60 solutions to the main problem. He corrected himself in the September 21 issue [1850c] and gave there all of the 92 solutions.

Gauss' involvement in the problem stems from his reading the problem in the *Illustrirte Zeitung* and is recounted in full in his correspondence with his astronomer friend H. C. Schumacher. The relevant letters were published in [Peters 1865].

Gauss wrote to Schumacher on September 1, proposing the problem to the latter and mentioning the *Illustrirte Zeitung* as his source. Gauss also remarked that the proposer had declared that there were 60 solutions, but that Gauss himself found 76.

Schumacher replied on September 4 (already!), clarifying that Gauss must have meant 76 as the number of solutions to the general problem, and not to the subproblem. Moreover, Schumacher asked Gauss if he would mind if Schumacher sent to Staunton,

editor of *Chess Chronicle*, the "correct" solution arrived at by Gauss, without mentioning the latter's name.

Gauss wrote back on September 12, noting that four of his proposed 76 solutions should be stricken, leaving 72 as the number of true solutions found by him; *but* he also noted that there might very well be more.

In his reply of September 24, Schumacher made a faulty attempt to determine the number of all possible solutions, arriving at crude estimates of 120 and 168.

Gauss wrote back on September 27, treating the problem at great length. He referred Schumacher to Nauck's correction in the September 21 *Illustrirte Zeitung* giving 92 as the correct number of solutions, and he repeated for Schumacher's benefit the 12 basic kinds of solutions as given in that article. But since Nauck had been wrong the first time in claiming that there were only 60 solutions, said Gauss, there was room to doubt the assertion that there are only 92, especially since Nauck offered no proof that there are not more. To resolve the question by a careful enumeration of solutions via trial and error, continued Gauss, would take only an hour or two. Apparently such inelegant work held little attraction for Gauss, for he does not seem to have carried it out, despite outlining in detail how to go about it. He continued on in the letter to reformulate the problem as an arithmetic one, and to relate it to the representation of complex numbers.

Schumacher concluded the correspondence on the problem on October 5 by making a few elaborations on Gauss' arithmetical reformulation.

The tenor of Gauss' letters leads a reader to surmise that in fact he devoted very little time to the problem and attached very little importance to it. He failed to solve the problem completely, but apart from having to correct himself from 76 solutions to 72, he acquitted himself well. He did not claim that the 76 or the 72 were all the solutions, and in fact he explicitly noted that there might be others. His customary caution, captured in his professional motto of *pauca sed matura* ("few works, but ripe ones") served him well.

We describe briefly the arithmetical reformulation Gauss gave. The task becomes the process of arranging the digits 1 through 8 in such an order, that adding 1 to the first of them, 2 to the second, ..., and 8 to the eighth, produces a collection of distinct sums; while at the same time adding 8 to the first, 7 to the second, ..., and 1 to the eighth, likewise produces a collection of distinct sums--though some of these numbers may coincide with numbers in the first collection.

The example Gauss offered Schumacher was the arrangement 15863724.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 5 | 8 | 6 | 3 | 7 | 2 | 4 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 2 | 7 | 11 | 10 | 8 | 13 | 9 | 12 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 5 | 8 | 6 | 3 | 7 | 2 | 4 |
| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| 9 | 12 | 14 | 11 | 7 | 10 | 4 | 5 |

We may interpret any permutation of the numbers 1 through 8 as a placement of queens, where for example 15863724 designates the position with queens et a1, b5, c8, and so on. Such a position automatically has exactly one queen in each row and each column. Two queens will be on the same diagonal exactly when they are as many rows apart as they are columns apart. An examination of the chessboard reveals that a diagonal rising to the right consists of squares whose row number and column number add up to the same amount; and a diagonal falling to the right is composed of squares whose row number and column number differ by the same amount. Excluding queens from lying on the same diagonal of the first kind corresponds to Gauss' first condition. That Gauss' second condition prevents two queens from lying on the same diagonal of the second kind is easily seen when we realize that Gauss added (9 - column number) to the row number instead of subtracting the column number from the row number; each of his results is larger by exactly 9 than the corresponding difference of column number and row number.

## LATER DEVELOPMENTS

Although Gauss' discussion with Schumacher took place in 1850, the correspondence did not appear in print until 1865. It is unlikely that others knew the contents of the letters before then. Meanwhile, the problem itself travelled and captured the fancy of chessplayers and mathematicians; unaware of Nauck's solution, they proceeded to publish their own independently discovered solutions.

Lionnet [1852] proposed the problem in its arithmetic form, noting the chess interpretation. The proposal appeared in the problem section of *Nouvelles Annales de Mathématiques*, but no solution ever appeared there. Instead, Lionnet's proposal was picked up and a complete solution given elsewhere by G. Bellavitis [1861]. Another independent complete solution was given by C. F. de Jaenisch [1862], while an incomplete solution by Solvyns was given in [Cretaine 1865]. In 1867 Parmentier and de la Noë also arrived at the complete solution, but an account of it did not appear until [Lucas 1880b]. Lionnet [1869] proposed the more general problem of n queens on an nxn board, and Bellavitis [1870] answered again with complete solutions for boards of several sizes. None of the individuals mentioned here referred to Bezzel, Nauck, or Gauss.

Günther [1874] accurately summarized and quoted at length the Gauss-Schumacher correspondence (though he was not aware of

of Bezzel's earlier proposal), noted mention of the problem by Jaenisch [1862], Lange [1865], and Natani [1867] and contributed his own method of solution via determinants, which he carried out only for the 4x4 and 5x5 boards.

## THE ERROR: APPEARANCE AND PROPAGATION

A follow-up to Günther's article by J. W. L. Glaisher [1874] began with a summary of Gauss' relation to the problem:

> *The problem ... was proposed by Nauck to Gauss, and formed the subject of a correspondence between the latter and Schumacher. Gauss, after finding the number to be 76 and then 72, ultimately arrived at 92, which has since been recognized as the correct solution. An interesting account of the history of the problem is given ... by Dr. Siegmund Günther ...* [1874, 457-458]

Three errors appear here. The first is the implication that the problem originated with Nauck, whereas in fact it had been proposed earlier by Bezzel ("Schachfreund"). The second is the distorted impression it conveys that Nauck directly communicated the problem to Gauss, while we have seen that Nauck merely published it in a newspaper Gauss happened to read. The third error is the statement that Gauss eventually gave the complete solution.

Glaisher's interest in the problem was to simplify Günther's approach and carry it through for boards of 6x6, 7x7, and 8x8, and he did not cite any works besides Günther's. In fact, no other sources prior to Glaisher gave the same version of the history as he did. It seems that he did not actually read the Gauss-Schumacher correspondence, since he failed to cite it; in fact, there would have been very little reason for him to seek it out, because of the copious quotations from it to be found already in Günther's paper. We must conclude that his sole source of information was Günther's paper. But that paper does not contain the errors, so we see here the generation of historical error. The error would be most easily explained if in fact Glaisher could not read German very well.

The main propagator of the error, however, was not Glaisher but the renowned number theorist and recreational mathematician Édouard Lucas. Lucas first wrote on the eight queens problem in [1880a] with further articles [1880b, 1886a, 1886b]. Moreover, he included it as the fourth chapter in the first volume of his book of mathematical recreations [1882]. There the following passage introduced the topic, and a comparison of the wordings leaves no doubt that he was paraphrasing Glaisher:

> *Ce problème a été proposé pour la première fois par Nauck a illustre Gauss ... cet question fut l'object*

*d'une correspondance entre ce dernier et l'astronome Schumacher. Après avoir trouvé 76, puis 72 solutions, Gauss trouva enfin le nombre de 92 solutions, qui a été reconnu definitevement pour le nombre exact.*

[1882, 59-60]

He went on to cite Günther, Glaisher, Bellavitis, and others, but not the pages of the Gauss-Schumacher correspondence. It is very likely that Lucas did not actually see the Günther article, as he almost surely did not see Bellavitis [1861]. For the latter he cited the same wrong page numbers as Bellavitis did in [Bellavitis 1870]. (See Ahrens [1910, 226] for details. To be fair, one should remark that in other instances Lucas exhibited painstaking concern for historical accuracy. See, for example, [Lucas 1887], where care was taken to reproduce entire title pages.)

Lucas' books were very popular and became the authoritative reference for mathematical recreations until the appearance of [Ahrens 1901]. It is no surprise then, that other writers who cited Lucas related the same history--e.g., Schubert [1892, 1899, 1900], Simon [1890], Parmentier [1884]. Other authors, however, picked up the misinformation directly from the Glaisher paper, as in the case of Ball in his book of mathematical recreations. The second edition [1892] of the book attributed the first complete solution of the problem to Gauss, while more recent editions [1974, 165] omit the remark and refer the reader to Ahrens [1910].

On the other hand, accurate history was related by Pein [1890], Sprague [1899, 44] (who read Pein and pointed out the error in Glaisher), and Ahrens [1910, 211-212].

## CONCLUSION

The real significance of all this lies in the events after Gauss: an English mathematician jumbled the facts, perhaps because his German was poor; a well-known French mathematician, whose historical delving on other topics was painstaking, probably accepted one of his sources without checking that source's sources or other references; but happily everything got straightened out in the end. Unfortunately, there is no official public record which can be set straight for posterity, so that in this case, as in so many others, false history continues to live beside the true. For example, a recent paper by Hoffman *et al.* [1969, 66] begins, "The problem of the m queens, originally introduced by Gauss (with m = 8), ..."

Mathematicians do not seem to take eagerly to the discipline of sorting out the history of their subject. A possible explanation is that history does not offer in the concern for detail that it shares with mathematics the same certainty about those details that can be afforded by mathematics. Still, it

behooves mathematicians to offer students historical perspectives on their researches, and a sense of scholarship compels a striving for those perspectives to be as accurate as possible.

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